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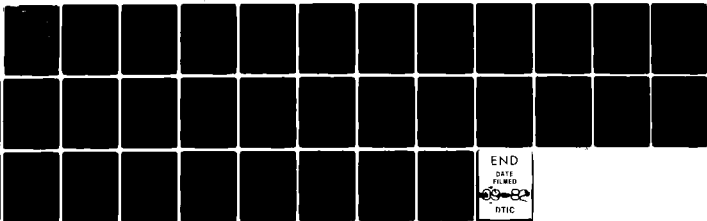
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PRACTICAL APPLICATIONS OF FRACTURE MECHANICS IN AIRCRAFT AND AEROSPACE STRUCTURAL PROBLEMS

by

J. J. Kacprzyński

National Aeronautical Establishment

OTTAWA
JUNE 1982

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National Research Council Canada, National Aeronautical Establishment.

1. Fracture mechanics.

- I. Kacprzynski, J.J.
- II. NRC, NAE LR-611

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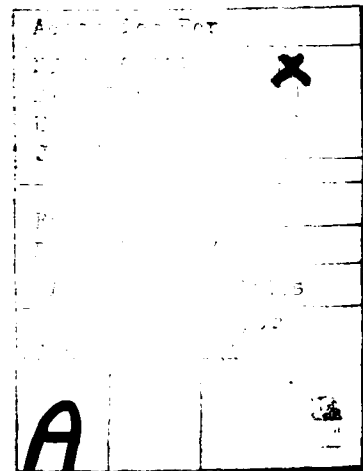
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APPLICATIONS PRATIQUES DES MÉCANISMES DE FRACTURE AU NIVEAU DES PROBLÈMES STRUCTURAUX DANS L'AVIATION ET L'AÉROSPATIAL

J.J. Kacprzyński



G.M. Lindberg
Director/Directeur

ABSTRACT

Two computer programs, CRACKS-IV and FLAGRO 4, used for the analysis of crack growth in aircraft and aerospace structures are reviewed. The merits and limitations of each program are described using practical numerical examples. Requirements for the next generation of computer programs are specified.

ABSTRAIT

Deux programmes d'ordinateur, CRACKS-IV et FLAGRO 4, utilisés pour l'analyse de croissance des fissurations sur des structures aérospaciales et d'avions sont revus. Les mérites et limitations de chaque programme sont expliqués, utilisant des exemples pratiques numériques. Les exigences pour les prochains programmes d'ordinateur sont spécifiées.

CONTENTS

	Page
ABSTRACT	(iii)
ILLUSTRATIONS	(v)
APPENDIX	(vi)
SYMBOLS	(vi)
1.0 INTRODUCTION	1
2.0 COMPARISON OF CRACKS IV AND FLAGRO 4	1
2.1 Stress Indensity Factors	1
2.2 Crack Growth Models	2
2.3 Lower Threshold Limit	3
2.4 Crack Growth Retardation	3
2.5 Material Data	3
2.6 Loads	4
2.7 Units	5
2.8 Special Features	5
3.0 NUMERICAL EXAMPLES	5
3.1 Through Crack in a Plate	5
3.2 Surface Cracks	6
3.3 Corner Crack	6
3.4 Cracks from a Hole	6
3.5 Retardation	6
3.6 Comments	7
4.0 THE FUTURE NEEDS OF FRACTURE MECHANICS	7
4.1 Materials	7
4.2 Units	8
4.3 Geometrical Capabilities	8
5.0 CONCLUSION	8
6.0 REFERENCES	8

TABLES

Table	Page
1 Crack Forms	11
2 Available Crack Growth Models	13

TABLES (Cont'd)

Table		Page
3	Interpolated Values of $\tan\alpha$ from a Table of Values of $\tan\alpha$ at Intervals of 5° with an Additional Value at $\alpha \geq 88^\circ$	13
4	Example of FLAGRO Material Data	14
5	Crack Growth Constants C for Alum 2219-T62, R = 0	14
6	Through Crack in a Plate, Alum 2219-T62, R = 0	15
7	Through Crack in a Plate, Alum 2219-T62, R = 0.5	15
8	Through Crack in a Plate, Alum 7075-T76, R = 0	16
9	Through Crack in a Plate, Alum 7075-T6, R = 0	16
10	Surface Crack, Alum 2219-T62, R = 0	17
11	Surface Crack, Alum 7075-T76, R = 0	17
12	Surface Crack, Alum 7075-T6, R = 0	18
13	Corner Crack, Alum 7075-T6, R = 0	18
14	A Through Crack from a Hole, Alum 7075-T6, R = 0	19
15	Two Cracks from a Hole, Alum 7075-T6, R = 0	19
16	Effect of Retardation on the Crack Growth in a Plate, CRACKS IV — Alum 2219-T62 (With Willenborg Retardation)	20
17	Effect of Retardation Parameter m of Wheeler Model	20
18	Effect of Retardation Parameter-Overload Ratio of Willenborg-Gallagher Model	20
19	Retarded Growth of a Through Crack in a Plate with Different Retardation Models, CRACKS IV — Alum 2219-T62 (Unretarded Forman Growth to Failure 23 Blocks)	21
20	A Through Crack in a Plate-Retardation in FLAGRO Program, Alum 2219-T62 (Retardation Coefficient m = 2.2)	21

ILLUSTRATIONS

Figure		Page
1	A Crack in Infinite Plate	23
2	Comparison of Crack Growth Models, Alum 2219-T62	24

ILLUSTRATIONS (Cont'd)

Figure		Page
3	Comparison of Crack Growth Models, Alum 7075-T76.	25
4	Comparison of Crack Growth Models, Alum 7075-T6.	26
5	The Effect of Stress Ratio R on Crack Growth.	27
6	A Through Crack.	28
7	A Surface Crack.	28
8	A Corner Crack.	28
9	A Crack from a Hole.	29
10	A Double Crack from a Hole.	29
11	A Through Crack in Alum 2219-T62 Plate Forman Growth with Wheeler Retardation.	30
Appendix A		31

SYMBOLS

Symbol	Definition
a	crack length or semi-length
c	depth of a crack
n	crack growth exponent
m	retardation parameter
p	exponent for closure factor
t	thickness
da/dN	crack growth rate
B	exponent for decreasing closure
C	crack growth constant
C_{f0}	closure factor at $R = 0$
C_{f-1}	closure factor at $R = -1$
K	stress intensity factor
N	the number of overload cycles required to achieve saturation
R	$\sigma_{\min}/\sigma_{\max}$ — stress ratio
β	stress intensity correcting factor
γ_1	effectiveness after one overload
σ	stress

PRACTICAL APPLICATIONS OF FRACTURE MECHANICS IN AIRCRAFT AND AEROSPACE STRUCTURAL PROBLEMS

1.0 INTRODUCTION

The December 22, 1969, crash of the F-111, caused by an extremely small initial crack is recognized as an event signalling the era of fracture mechanics in the aircraft and aerospace industries. This event showed that small flaws could cause catastrophic failures. It gave a tremendous boost to fracture mechanics. Terms such as fracture toughness, slow crack growth and scatter factors, became part of everyday vocabulary. Many methods and computer programs for crack growth prediction have been developed. While most methods are unclassified, many computer programs applying these methods have restricted distributions.

In spite of the tremendous progress in fracture mechanics, the accurate prediction of crack growth is extremely difficult. Fracture mechanic methods are not exact, stress intensity calculations are approximate, loads are not known accurately, material and crack growth data are approximate, state of stress (plane stress, plane strain or mixed) is difficult to determine, the methods of integration of crack growth data are approximate and finally, the effects of variable amplitude loads causing crack growth retardation or acceleration are not completely understood. Even for the same load spectrum, the same crack growth model, and the same crack growth constants, different programs may give different predictions. In many cases it is difficult to assess the accuracy of the results. Consequently, the trend is to compare calculations against results from standard programs. For defence work, the aircraft industry in the United States uses the USAF CRACKS programs as standards. The latest version, CRACKS IV, although unclassified, is restricted in distribution. The only Canadian report describing it in detail is a defence proprietary restricted NRC-NAE report¹.

The civilian aerospace industry standard is a program called FLAGRO, which is distributed internationally by COSMIC².

In the present paper, a comparison is made of CRACKS IV and FLAGRO 4, and their capabilities and limitations are discussed. Several numerical examples are included. A new program, which could be used by both the aeronautical and the aerospace industries, is discussed. It is anticipated that this program, when completed, can be freely distributed to Canadian users.

2.0 COMPARISON OF CRACKS IV AND FLAGRO 4

Linear Elastic Fracture Mechanics (LEFM) is based on the concept of a stress intensity factor describing the state of stress in the vicinity of the crack tip. Crack growth is related to the stress intensity. CRACKS IV and FLAGRO 4 will be compared in terms of the stress intensity factors generated, the models of crack growth and retardation available, and the ability of the programs to handle different crack types, loading conditions, and other special features.

2.1 Stress Intensity Factors

LEFM assumes that the propagation of an existing crack is governed by the stress intensity at the crack tip. Both FLAGRO 4 and CRACKS IV use a stress intensity factor for a through crack in an infinite plate under tension (Fig. 1). Stress intensity factors for other specimens and crack configurations are derived from this reference value. The stress intensity factor may be used in two forms, FLAGRO 4 uses the ASTM form;

$$K_I = \sigma\sqrt{\pi a}$$

while CRACKS IV uses both the above ASTM form and the so-called NASA form;

$$K_I = \sigma\sqrt{a}.$$

Each of these representations requires different crack growth data. Most published data is for the ASTM form of stress intensity factor and therefore selecting $K_I = \sigma\sqrt{a}$ in CRACKS IV may give misleading results if used inadvertently.

For other geometries and crack forms, the stress intensity factor is calculated from the above values using a correction factor. Hence, for a problem other than a through crack in an infinite plate, the ASTM stress intensity is given by the expression;

$$K_I = \sigma\sqrt{\pi a} \beta$$

where β is a correction factor.

The crack forms covered by FLAGRO 4 and CRACKS IV are not identical, as shown in Table 1.

Table 1 indicates that CRACKS IV is limited to the analysis of cracks in thin plates under tension. The capabilities of FLAGRO 4 are much wider, and include the bending state of stress for surface and corner cracks, edge beam cracks, as well as through and corner cracks in loaded or open lugs and holes. These capabilities are very important in aircraft, helicopters and space structures.

In CRACKS IV correction factors can be presented in a tabular form, as a function of crack length, which allows crack growth calculations to be carried out for any crack geometry for which the stress intensity factors are known. Up to two tables of correction factors can be used. CRACKS IV can also be used to calculate crack growth in compact tension specimens, which is useful since it allows one to compare numerical predictions with experimental data.

2.2 Crack Growth Models

Over the years many empirical models of crack growth have been developed and some of these have been incorporated in FLAGRO 4 and CRACKS IV. The available models are listed in Table 2 (see also Appendix A).

In the case of a part-through surface crack or a corner crack, FLAGRO 4 allows different crack growth constants to be specified for the depth and width directions. The program also recommends the use of different fracture toughness values in these directions — the plane strain value for the depth direction and the plane stress value for the width direction and for through cracks. CRACKS IV uses the same crack growth constants in both directions, together with one value of fracture toughness, and one value of cut-off stress intensity.

As indicated in Table 2, both FLAGRO and CRACKS will accept crack growth rate data in a tabular form, which consists of values of da/dN presented as a function of ΔK . For CRACKS IV, a table is used for one unspecified stress ratio R only. The program finds the interpolated value of da/dN for any crack length. Linear interpolation is done not between da/dN values, but between their decimal logarithms, which gives better accuracy. In FLAGRO 4, several tables of da/dN are given for different values of crack length a and stress ratio R , and the program performs a double linear interpolation, in respect to R and a .

In order to illustrate the difference between the interpolation methods used by FLAGRO and CRACKS, an example was calculated (Table 3) of the interpolation of $\tan \alpha$ from the exact values calculated between 35° and 85° at intervals of 5° with the additional value at 88° . The interpolated values were calculated between these points. The results show the superiority of the technique used in CRACKS IV. The interpolation errors partly explain the differences in results calculated by the two programs.

2.3 Lower Threshold Limit

CRACKS IV allows the variation of the threshold stress intensity range (ΔK_{th}) with stress ratio R to be defined by introducing:

$$\Delta K_{th} = \Delta K_{th} (1 - A \cdot R)$$
$$R = 0$$

where A is a constant.

Unfortunately, very few experimentally determined values of A exist.

2.4 Crack Growth Retardation

The importance of retardation or acceleration due to variable amplitude loads have been recognized for many years, and many retardation models have been developed¹. Some of these are included in CRACKS IV, and may require experimental constants, as described below.

- a. Wheeler model — requires input of the constant m (Ref. 7)
- b. Willenborg model — no additional constants (Ref. 8)
- c. Willenborg-Gallagher model — requires input of the overload ratio m for total retardation (Ref. 9)
- d. Crack closure model — allows both retardation and acceleration to be evaluated (Ref. 10). It also accepts compressive stresses (stress ratio R may be negative — all other models set negative R to zero). Six experimental values are required;

C_{fo} — closure factor at $R = 0$

C_{f-1} — closure factor at $R = -1$

p — exponent for closure factor

B — exponent for decreasing closure

γ_1 — effectiveness after one overload

N_{sat} — the number of overload cycles required to achieve saturation.

Unfortunately these data are known only for two materials, 2219-T851 aluminum and Ti-6AL-4V.

While the FLAGRO 4 manual states that the Willenborg retardation model is used, it appears that Wheeler's model is used instead, and it also appears to be used incorrectly. This feature of FLAGRO 4 should therefore be used with caution.

CRACKS IV allows calculations using retardation to be repeated automatically without retardation.

2.5 Material Data

Depending on the crack growth model and retardation model used, some of the following material data may be needed:

- yield strain
- lower threshold stress intensity with R effects
- cut-off stress intensity factor
- critical toughness (for plane stress or plane strain or both)
- crack growth constants
- crack retardation constants.

FLAGRO 4 supplies material and crack growth data for the Collipriest model for 18 aluminum alloys, 11 titanium alloys, 15 steels, 4 heat resistant alloys, and 6 non-ferrous materials. Data is also supplied for welded joints in 24 materials, data for diffusion-bonded joints in four materials and Forman crack growth data for annealed Ti-6Al-4V. The user must specify the material code number listed in the manual and FLAGRO automatically sets the data. For example, the material data for three aluminum alloys are given in Table 4. The FLAGRO capability for automatic supply of the basic material data is extremely valuable, since it saves time and avoids errors. It is unfortunate that standardized data are still not available for many of the alloys used in the aircraft industry.

The FLAGRO manual recommends the use of plane strain values of fracture toughness for the depth direction, and plane stress values for the width direction. This is demonstrated in the manual for 7075-T6 (material CODE 112). FLAGRO also has the capability for manual input of material data and the retardation constant must be input manually.

The material data, crack growth data and retardation data must be input manually to CRACKS, since a default program will introduce constants for Forman's model for 7075 aluminum.

2.6 Loads

In FLAGRO loads have to be specified either in a stress form (σ max, σ min in KSI and number of cycles) or in force (moment) form with an additional function transforming them to stress.

The specified stresses (forces) constitute a block of loads and the crack growth calculations are performed for the specified number of blocks or until the cut-off value of either the stress intensity factor or the length crack are exceeded.

In CRACKS there are further options. Loads have to be specified as stresses (in psi or KSI) for all cases except the compact tension specimens, where forces must be specified. Three forms of load are available:

1. maximum stress (force) — minimum stress (force).
2. R-DELTA (R-stress ratio and DELTA being the difference of stresses, σ max — σ min, or forces).
3. Mean-Alt, where Mean is the mean stress (force) and Alt is alternating stress (force).

This information, together with the number of cycles, is used to define mission segments. These can be arranged in an arbitrary way to create a block which is then applied repeatedly as required. Both in FLAGRO and in CRACKS (with the exception of the analysis with crack closure retardation) the compressive loads are set to zero.

2.7 Units

As an input, FLAGRO requires stresses in KSI. The crack growth constants for the Collipriest, Paris and Forman models must be calculated from the formula;

$$C_{\text{input}} = \frac{C_p}{(10^{3n+6})} \quad (\text{micro inch for } K \text{ in KSI } \sqrt{\text{inch}})$$

where C_p is a physical crack growth constant in inches for K in psi (in)^{1/2} and n is the appropriate exponent.

When tabular crack growth data is used, FLAGRO requires different units, da/dN in inches and ΔK in psi(in)^{1/2}.

In CRACKS, stresses may be defined in psi or KSI, with a proper value of the crack growth constant in inches and taking into account the form of K (ASTM or NASA) used.

2.8 Special Features

FLAGRO allows the effects of the compressive residual stress existing at the edge of cold-worked holes to be taken into account, (Ref. 11). This is potentially useful, but too few experimental data exist for it to be used effectively.

CRACKS has a capability for storing the calculated results for future restart. This is extremely useful with complex calculations involving, for example, life estimates of an aircraft.

3.0 NUMERICAL EXAMPLES

Examples are provided for the three materials described in Table 4. The crack growth constants supplied for the Collipriest model in FLAGRO were transformed to standard units; da/dN in inches, K in psi(in)^{1/2}, and the constants for the Paris and Forman models were obtained by curve fitting. Table 5 illustrates the difference in constant values for 2219-T62 aluminum necessitated by unit requirements. These differences may lead to accidental errors.

A comparison of the crack growth model for the three alloys at $R = 0$ is presented in Figures 2-4. The agreement between the Collipriest and Forman models is very good. When the Paris model is used with only one set of constants, the agreement deteriorates at high ΔK (Fig. 2). A better agreement could be obtained using two sets of constants, but since this was not possible with FLAGRO, it was not attempted.

The effect of stress ratio R on crack growth is shown in Figure 5. The calculated examples of unretarded crack growth were for the same load — one block is determined as follows:

for $R = 0$	$\sigma_{\text{max}} = 10 \text{ KSI}, \sigma_{\text{min}} = 0, 1000 \text{ cycles}$
	$\sigma_{\text{max}} = 15 \text{ KSI}, \sigma_{\text{min}} = 0, 100 \text{ cycles}$
and for $R = 0.5$	$\sigma_{\text{max}} = 20 \text{ KSI}, \sigma_{\text{min}} = 10 \text{ KSI}, 1000 \text{ cycles}$
	$\sigma_{\text{max}} = 30 \text{ KSI}, \sigma_{\text{min}} = 15 \text{ KSI}, 100 \text{ cycles.}$

3.1 Through Crack in a Plate

The growth of a through crack in a 2219-T62 plate, Figure 6, was calculated using CRACKS IV and FLAGRO 4. The number of blocks to failure are shown in Table 6. The stress intensities are

the same in both programs, so the differences in blocks to failure are due solely to the crack growth models or numerical techniques used.

Crack growth rates can be presented in the form of a table consisting of values of $(da/dN)_i$ versus ΔK_i and which in the present case are generated from the models of either Collipriest, Forman or Paris.

In the present case, the agreement is pretty good between both programs. The differences in tabular growth in FLAGRO and CRACKS are caused by the different techniques of interpolation. The differences between Collipriest and other models are caused by the different representations of crack growth. Small differences in calculated lives to failure are noted for 2219 aluminum at $R = 0.5$ in Table 7 and for two other aluminum alloys at $R = 0$ in Tables 8 and 9.

3.2 Surface Cracks

A surface crack (Fig. 7) in a plate 20" wide, 0.1" thick was studied. Initial crack size was assumed to be $a = c = 0.05''$. The same crack growth constants were used as in the previous case in Section 3.1. Calculations were performed for $R = 0$ and the results are shown in Tables 10-12. The number of load blocks for transition to a through crack and for failure are given. For this case the agreement is not so good, due to small differences in stress intensity factors and crack growth models. FLAGRO uses different values of the crack growth constant C and critical stress intensity in the depth and width directions until transition. Accordingly, results from FLAGRO and CRACKS, using Forman's model for 7075-T6 aluminum, differ significantly.

3.3 Corner Crack

A corner crack (Fig. 8) was studied in a plate 10" wide and 0.1" thick. The initial crack size was $a = c = 0.05''$. The geometry represents one half of the surface crack shown in Figure 7. The calculations were performed only with FLAGRO, because CRACKS does not have a corner crack capability. The results for 7075-T6 aluminum at $R = 0$ are given in Table 13. A comparison of Tables 12 and 13 shows that a corner crack cannot be treated as a half of a surface crack.

3.4 Cracks from a Hole

Cases of a single and a double crack from a hole (Figs. 9, 10) were studied. The results for 7075-T6 aluminum at $R = 0$ are shown in Tables 14 and 15. The agreement is quite good, which may be explained by the identical Bowie stress intensity factors used in both programs.

3.5 Retardation

Retardation was studied for a through crack in a plate (Fig. 6) under the following block loading:

	$\sigma_{\max} = 20 \text{ KSI}, \sigma_{\min} = 0, \quad 100 \text{ cycles}$
block of loads	$\sigma_{\max} = 15 \text{ KSI}, \sigma_{\min} = 0, \quad 2000 \text{ cycles}$
	$\sigma_{\max} = 10 \text{ KSI}, \sigma_{\min} = 0, \quad 3000 \text{ cycles}$

The results obtained using the Willenborg retardation model and different crack growth models for a 2219-T62 aluminum plate are shown in Table 16. In this case retardation extends the life by nearly 3 times.

The effects of the retardation parameter m in Wheeler's model are shown in Table 17. The effects of the retardation parameter — overload ratio of Willenborg-Gallagher model are shown in Table 18. A problem is that in practical calculations, it is difficult to estimate the retardation parameters. If the proper retardation parameters are available for each model then practically the same results may be obtained as shown in Table 19. The results obtained using Wheeler's retardation model

with FLAGRO for different crack growth models are shown in Table 20. The results obtained do not differ very much from those obtained from CRACKS, as shown in Figure 11.

3.6 Comments

These simple examples show that the two programs may produce widely differing results for some cases, even for nominally identical test conditions. For more complex problems the scatter is even greater, and it is extremely difficult to know which results are most accurate. In practice the analyst usually has access to only one program, and may be limited by time and cost to perform only one calculation. The program used must, therefore, be well understood, free of errors, and must be supplied with reliable information on material properties and applied loads. If any of these conditions are not met, the chance of producing meaningful data is small.

4.0 THE FUTURE NEEDS OF FRACTURE MECHANICS

It is obvious that a new program for fracture mechanics calculations would be desirable that combines the best features of CRACKS and FLAGRO. This program should cover the most popular crack growth models used at present in aircraft and aerospace industries, namely:

- Paris (with two sets of constants as in CRACKS)
- Forman
- Walker
- Collipriest
- Tabular form with effects of R as in FLAGRO

In the past much effort has been spent on the development of new crack growth models and new retardations models. These models aim to provide a simple mathematical representation of experimental crack growth data. However the tabular growth model of experimentally determined da/dN versus ΔK for several R is probably more accurate than any mathematical model. One problem is the large amount of experimental data required, but this can be resolved by providing data on a magnetic tape or disk.

Retardation is extremely important and has been recognized by the aeronautical industry. However, because of insufficient experimental data, retardation has not been widely employed in the analysis of aeronautical problems. Only one retardation model (Willenborg) does not require experimental data. It seems that an extensive experimental effort is required to determine the retardation parameters for basic materials. A new generation program should cover the following retardation models:

- Wheeler
- Willenborg
- Willenborg-Gallagher
- Crack closure.

4.1 Materials

FLAGRO has the extremely useful feature of providing material and crack growth data for the Collipriest model for the most common materials used in aerospace industries. A list of about 40-50 basic materials is required for the aeronautical industry, together with elastic property data, crack growth data and retardation data. The program should be able to repeat calculations automatically, using appropriate data for each model in turn.

4.2 Units

In order to avoid accidental errors, the program should employ similar units for all options. Also, to avoid confusion, the stress intensity factor should be used only in one form, e.g. (ASTM)

$$K = \sigma\sqrt{\pi a}$$

At present most of the data are in conventional units (inch, psi) and therefore it would be convenient to retain these units, however an option for automatic use of SI units should exist.

4.3 Geometrical Capabilities

Geometry capabilities should be at least similar to FLAGRO. The bending effects are also important in aeronautical structures. They should be included in the surface crack, corner crack and edge beam cases. The program should have the capability of analysing edge cracks in a plates and lugs. Cold worked holes also require attention.

5.0 CONCLUSION

The presented comparison of the two most sophisticated programs used by the aeronautical and aerospace industries indicate that the existing numerical capabilities are not totally satisfactory. Some deficiencies can be addressed through the development of a new program which would expand on the best features of CRACKS and FLAGRO. This is a very large and tedious process, but it should be done in order to provide Canadian industry with a reliable tool. Some efforts to generate such a program has been already undertaken at NAE, but at present it is difficult to predict when the work will be completed.

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9. Gallagher, J.P.
Hughes, T.F. *The Influence of Yield Strength on Crack Growth Rate Delay in 4340 Steel.*
Air Force Flight Dynamics Laboratory, AFFDL-TR-74-27, March 1974.
10. Bell, P.D.
Creager, M. *Crack Growth Analysis for Arbitrary Spectrum Loading.*
AFFDL-TR-74-129, October 1974.
11. Chang, J.B. *Prediction of Fatigue Crack Growth at Cold-Worked Fasteners Holes.*
J. Aircraft, Vol. 14, 1977, pp. 903-908.

TABLE 1
CRACK FORMS

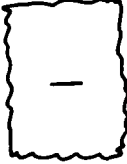
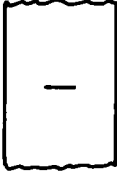
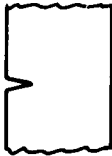


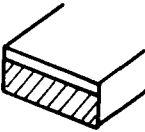
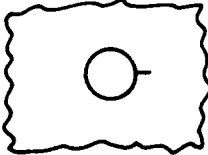
Form of a Crack		FLAGRO 4	CRACKS IV
a through crack in an infinite plate		YES	YES
a crack in a finite width plate		YES	YES
edge through crack		YES	NO
surface crack		YES in tension and in bending	YES in tension only
corner crack		YES in tension and in bending	NO
edge beam		YES	NO
a through crack from a hole		YES	YES

TABLE 1 (Cont'd)

CRACK FORMS (Cont'd)

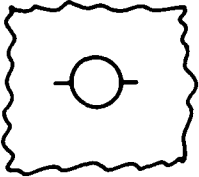
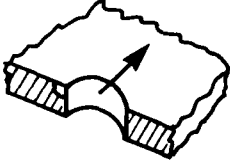
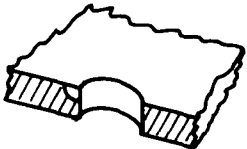
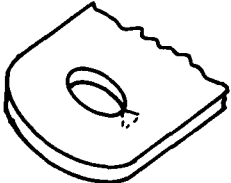
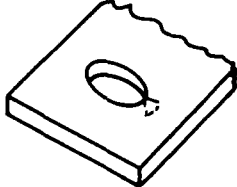
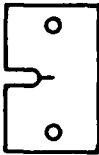
Form of a Crack	FLAGRO 4	CRACKS IV
double crack from a hole 	YES	YES
corner crack at a loaded hole 	YES	NO
corner crack at an open hole 	YES	NO
corner crack at open or loaded semicircular lug 	YES	NO
corner crack at open or loaded rectangular lug 	YES	NO
compact tension specimen 	NO	YES a. ASTM geometry b. Grumman geometry
tabular correction factor β as a function of crack length — it may represent any geometry	NO	YES up to two tables

TABLE 2

AVAILABLE CRACK GROWTH MODELS

Model (Data Format)	FLAGRO	CRACKS IV
Paris (Ref. 4)	YES*	YES**
Forman (Ref. 5)	YES*	YES
Walker (Ref. 3)	NO	YES
Collipriest (Ref. 6)	YES	NO
Tabular Format $\frac{da}{dN} = f(\Delta K)$	YES for several R	YES for one unspecified R

* Errors were found in the programs supplied to NRC. Analysts should check their programs carefully before use.

** CRACKS allows the Paris model to be used with one or two sets of crack growth constants. In the case of two sets of constants, we have

$$C_1, n_1 \text{ for } \Delta K \leq \Delta K_{\text{common}}$$

$$C_2, n_2 \text{ for } \Delta K \geq \Delta K_{\text{common}}$$

This capability provides a better approximation of the crack growth curve than in the case of only one set.

TABLE 3

INTERPOLATED VALUES OF $\tan \alpha$ FROM A TABLE OF VALUES OF $\tan \alpha$ AT
INTERVALS OF 5° WITH AN ADDITIONAL VALUE AT $\alpha \geq 88^\circ$

α°	37	42	47	52	57	62	67	72	77	82	87
$\tan \alpha$ (exact)	0.754	0.900	1.07	1.28	1.54	1.88	2.36	3.08	4.33	7.12	19.1
CRACKS	0.753	0.900	1.07	1.28	1.54	1.89	2.37	3.11	4.41	7.51	21.1
FLAGRO	0.756	0.903	1.08	1.29	1.55	1.90	2.39	3.14	4.51	7.97	22.9

TABLE 4

EXAMPLE OF FLAGRO MATERIAL DATA

CODE	106	108	112
material	AL 2219-T62	AL 7075-T76	AL 7075-T6
yield stress, KSI	36.	65.	65.
threshold stress intensity, KSI(in) ^{1/2}	3.5	3.	3.
critical stress intensity:			
— for part-through crack:			
in depth direction, KSI(in) ^{1/2}	35.	50.	33.
in width direction, KSI(in) ^{1/2}	35.	50.	73.
— for through crack, KSI(in) ^{1/2}	35.	50.	73.
cut-off stress intensity:			
— for part-through crack:			
in depth direction, KSI(in) ^{1/2}	35.	50.	33.
in width direction, KSI(in) ^{1/2}	35.	50.	73.
— for through crack, KSI(in) ^{1/2}	35.	50.	73.
Collipriest constant for FLAGRO			
C	0.008	0.0063	0.0436
n	2.79	3.	2.528

TABLE 5

CRACK GROWTH CONSTANTS C FOR ALUM 2219-T62, R = 0

	FLAGRO	CRACKS	
Units	micro inch for K in KSI(in) ^{1/2}	inch for K in KSI(in) ^{1/2}	inch for K in psi(in) ^{1/2}
Collipriest	0.008	8. · 10 ⁻⁹ *	0.341 · 10 ⁻¹⁶ *
Paris	3.835 · 10 ⁻⁴	3.835 · 10 ⁻¹⁰	1.806 · 10 ⁻²²
Forman	279.76	2.7976 · 10 ⁻⁷	3.2718 · 10 ⁻¹²

* no Collipriest model in CRACKS, constant given for comparison.

TABLE 6

THROUGH CRACK IN A PLATE

ALUM 2219-T62, R = 0

Model	Number of Blocks to Failure	
	FLAGRO 4*	CRACKS IV
Collipriest	91	—
Forman	75	76
Paris	75	76
tabular for Collipriest	87	98
tabular for Forman	73	77
tabular for Paris	71	77

* corrected for the proper execution of Forman and Paris models.

TABLE 7

THROUGH CRACK IN A PLATE

ALUM 2219-T62, R = 0.5

Model	Number of Blocks to Failure	
	FLAGRO 4*	CRACKS IV
Collipriest	78	—
Forman	64	65
tabular for Forman	62	66

* corrected for the proper execution of Forman model.

TABLE 8

THROUGH CRACK IN A PLATE

ALUM 7075-T76, R = 0

Model	Number of Blocks to Failure	
	FLAGRO 4*	CRACKS IV
Collipriest	77	—
Forman	86	87

* corrected for the proper execution of Forman model.

TABLE 9

THROUGH CRACK IN A PLATE

ALUM 7075-T6, R = 0

Model	Number of Blocks to Failure	
	FLAGRO 4*	CRACKS IV
Collipriest	42	—
Forman	33	28

* corrected for the proper execution of Forman model.

TABLE 10

SURFACE CRACK
ALUM 2219-T62, R = 0

Model		Number of Blocks	
		FLAGRO 4*	CRACKS IV
Collipriest	transition	143	—
	failure	187	
Forman	transition	161	125
	failure	208	162

* corrected for the proper execution of Forman model.

TABLE 11

SURFACE CRACK
ALUM 7075-T76, R = 0

Model		Number of Blocks	
		FLAGRO 4*	CRACKS IV
Collipriest	transition	110	—
	failure	150	
Forman	transition	140	207
	failure	181	249
tabular Collipriest	transition	107	157
	failure	146	198
tabular Forman	transition	148	237
	failure	187	279

* corrected for the proper execution of Forman model.

TABLE 12

SURFACE CRACK
ALUM 7075-T6, R = 0

Model		Number of Blocks	
		FLAGRO 4*	CRACKS IV
Collipriest	transition	43	—
	failure	69	
Forman	transition	19	48
	failure	43	70
tabular for Collipriest	transition	44	62
	failure	69	90
tabular for Forman	transition	36	60
	failure	59	83

* corrected for the proper execution of Forman model.

TABLE 13

CORNER CRACK
ALUM 7075-T6, R = 0

Model		Number of Blocks
		FLAGRO 4*
Collipriest	transition	33
	failure	49
Forman	transition	26
	failure	39
tabular Collipriest	transition	36
	failure	53
tabular Forman	transition	29
	failure	43

* corrected for the proper execution of Forman model.

TABLE 14

A THROUGH CRACK FROM A HOLE

ALUM 7075-T6, R = 0

Model	Number of Blocks	
	FLAGRO 4*	CRACKS IV
Collipriest	105	—
Forman	96	87
tabular Collipriest	104	96
tabular Forman	98	88

* corrected for the proper execution of Forman model.

TABLE 15

TWO CRACKS FROM A HOLE

ALUM 7075-T6, R = 0

Model	Number of Blocks	
	FLAGRO 4*	CRACKS IV
Collipriest	86	—
Forman	77	70
tabular Collipriest	85	79
tabular Forman	81	70

* corrected for the proper execution of Forman model.

TABLE 16

EFFECT OF RETARDATION ON THE CRACK GROWTH IN A PLATE

CRACKS IV — ALUM 2219-T62
(With Willenborg Retardation)

	Forman	Paris	Tabular for Forman
retarded growth	64*	95	66
unretarded growth	23	28	24

* number of blocks to failure.

TABLE 17

EFFECT OF RETARDATION PARAMETER m OF WHEELER MODEL

m	0 no retardation	1.3	1.7	1.9	2.1	2.15	2.2
number of blocks to failure	23	42	51	57	53	64	66

TABLE 18

EFFECT OF RETARDATION PARAMETER-OVERLOAD RATIO
OF WILLENBORG-GALLAGHER MODEL

m	no retardation	1.5	1.6	1.63	1.7	1.8	2
number of blocks to failure	23	90	69	66	58	43	36

TABLE 19

RETARDED GROWTH OF

A THROUGH CRACK IN A PLATE WITH DIFFERENT RETARDATION MODELS

CRACKS IV — ALUM 2219-T62

(Unretarded Forman Growth to Failure 23 Blocks)

Wheeler $m = 2.2$	Willenborg	Willenborg-Gallagher $m = 1.63$	Crack Closure for Tabular Forman Growth With Retardation Data for ALUM 2219-T851
66	64	66	34*

* incorrect retardation parameters.

TABLE 20

A THROUGH CRACK IN A PLATE-RETARDATION IN FLAGRO* PROGRAM

ALUM 2219-T62

(Retardation Coefficient $m = 2.2$)

Model	Collipriest	Paris	Forman	Tabular
unretarded growth	25	27	22	22
retarded growth { corrected program	84	83	78	76
original program (incorrect results)	27	32	24	23

* corrected for the proper execution of Forman and Paris models.

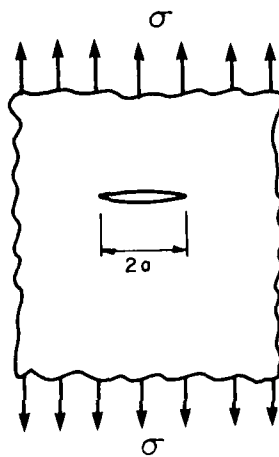


FIG. 1: A CRACK IN INFINITE PLATE

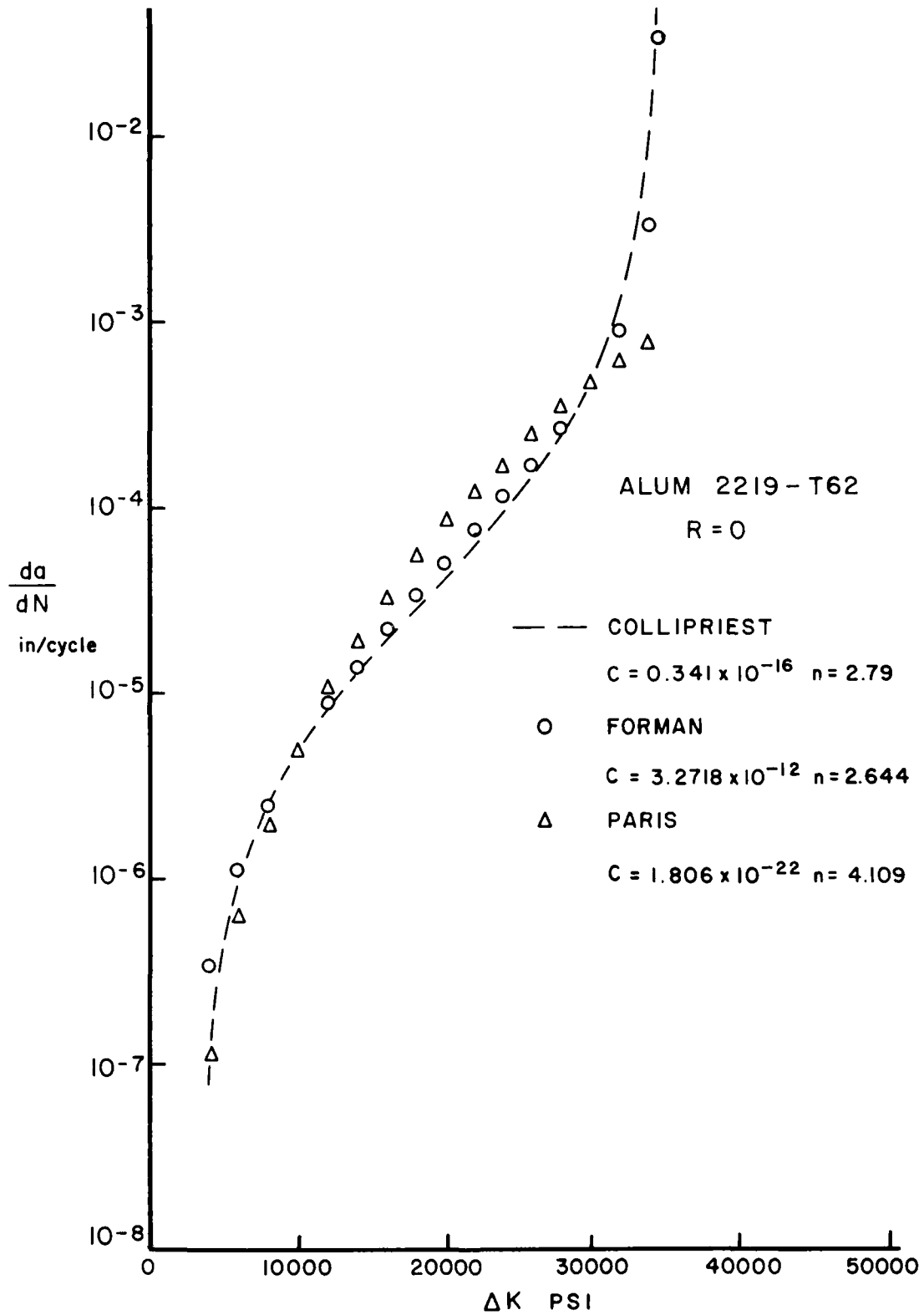


FIG. 2: COMPARISON OF CRACK GROWTH MODELS
ALUM 2219-T62

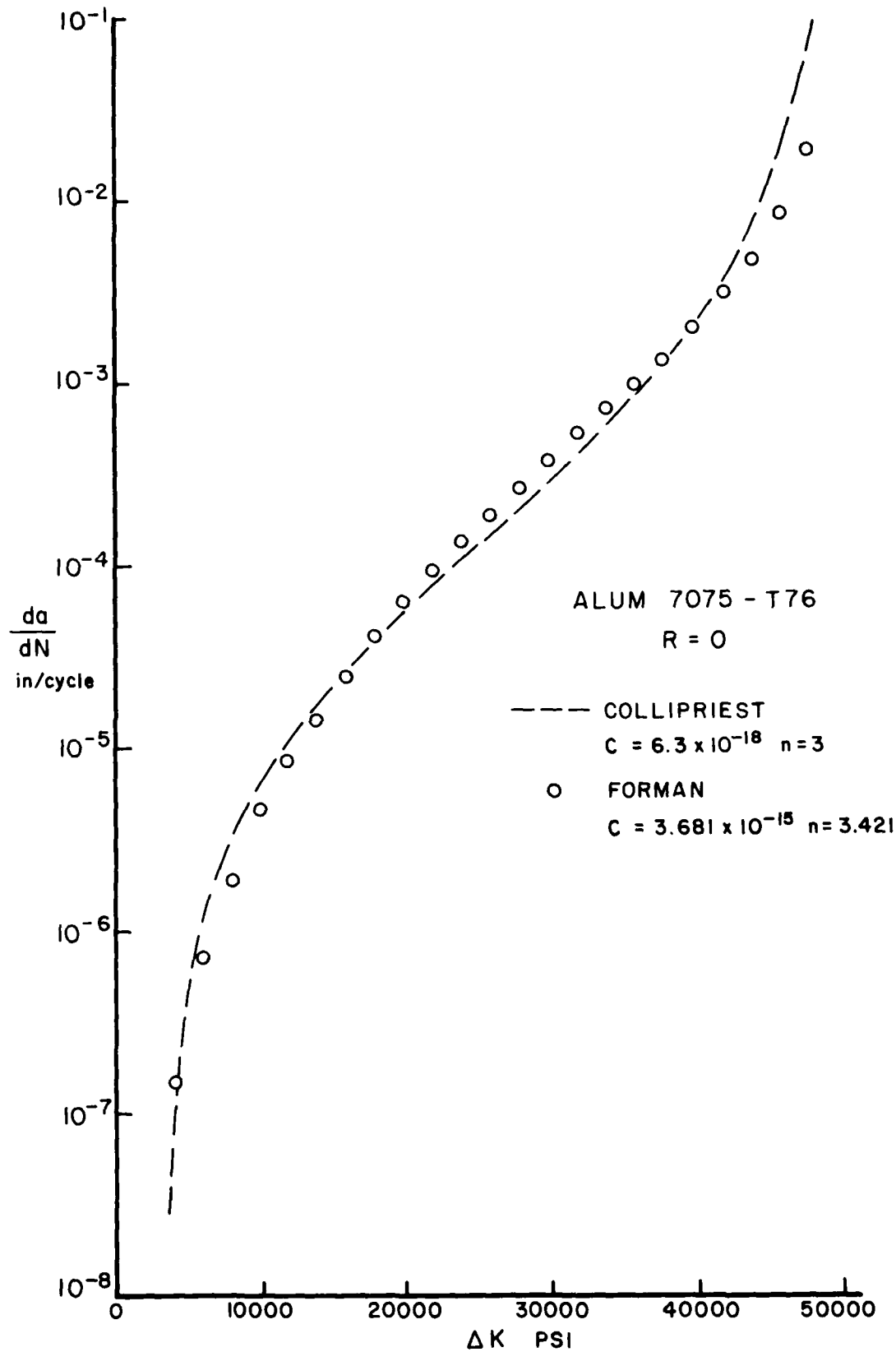


FIG. 3: COMPARISON OF CRACK GROWTH MODELS
ALUM 7075-T76

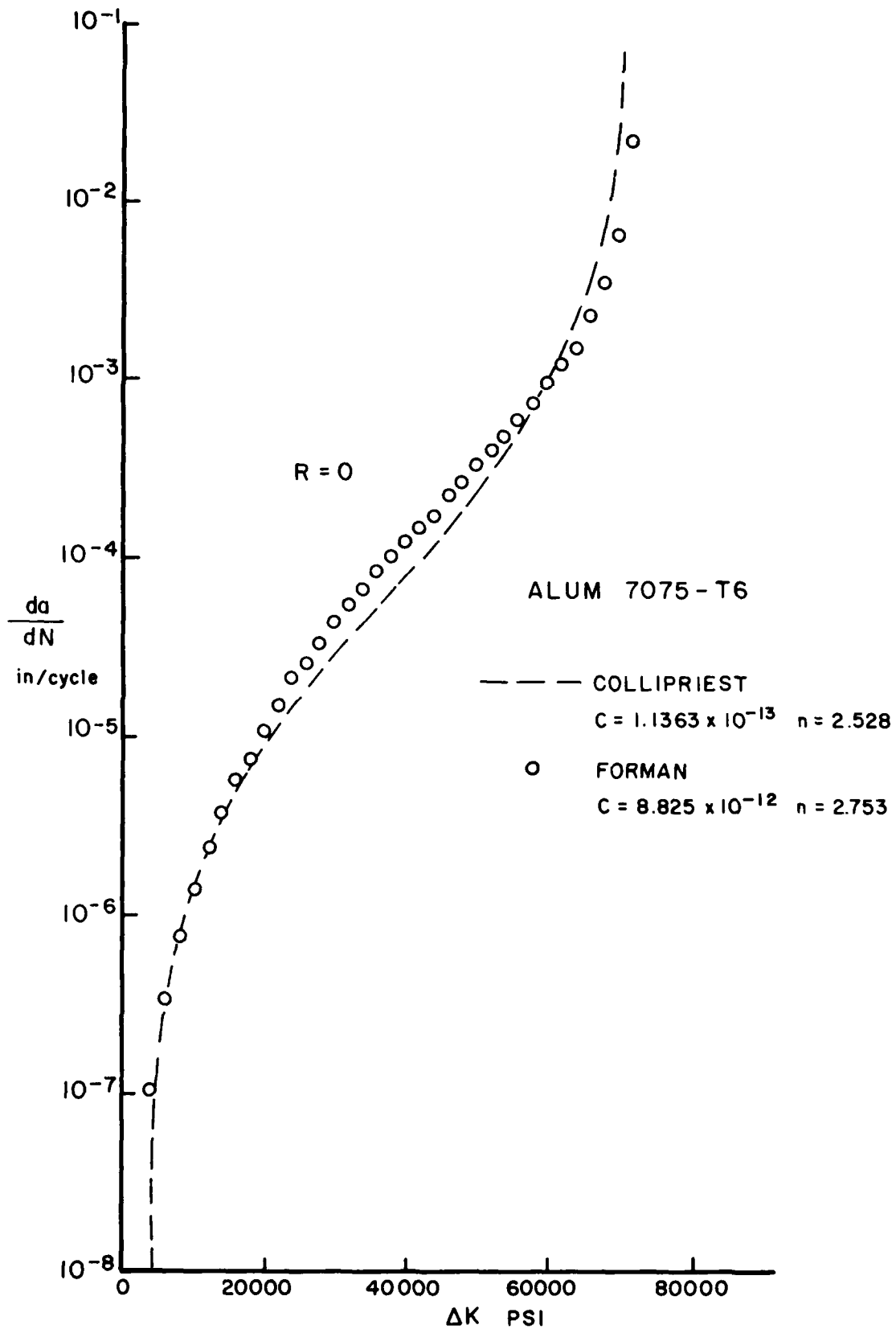


FIG. 4: COMPARISON OF CRACK GROWTH MODELS
ALUM 7075-T6

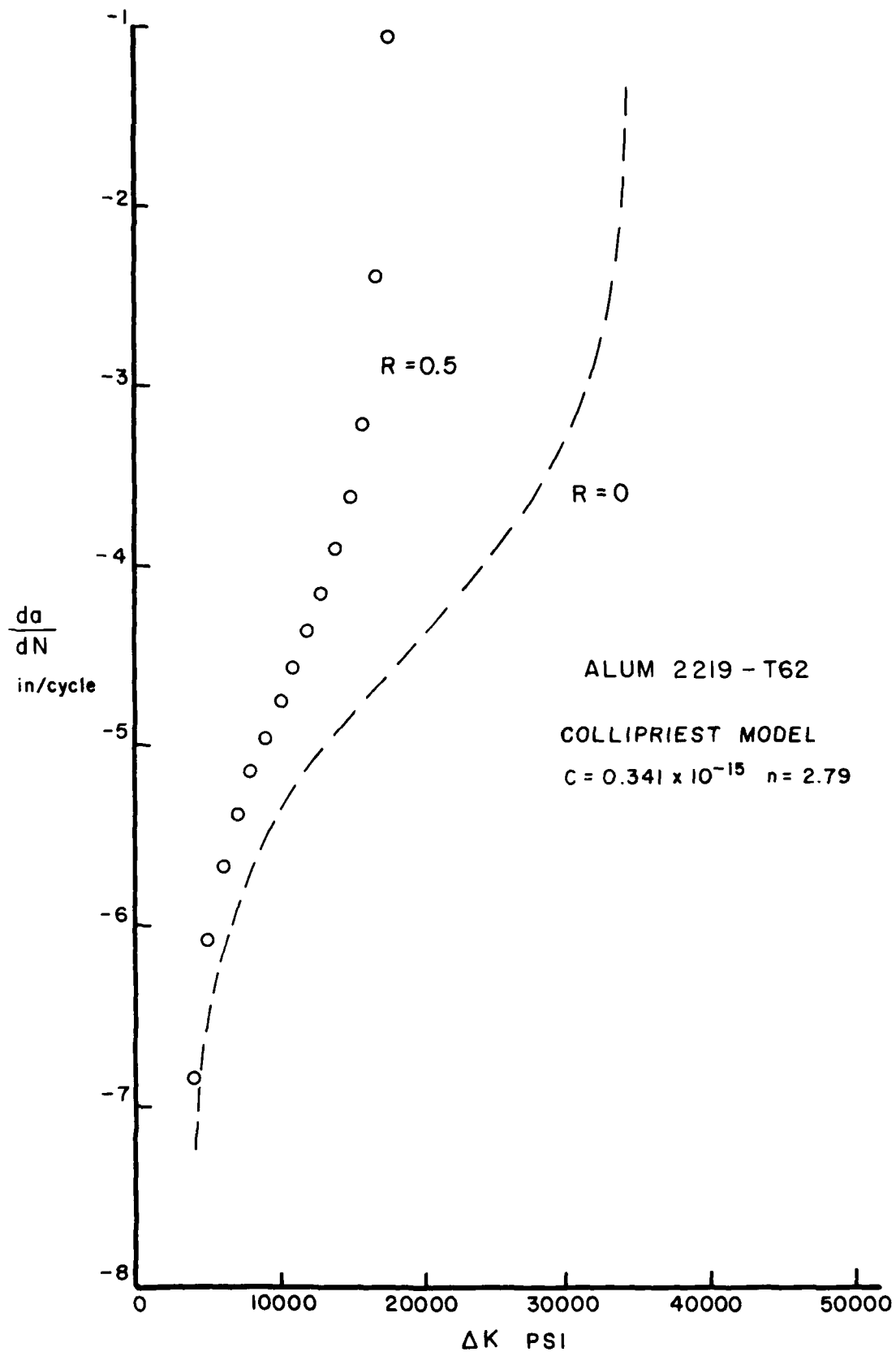


FIG. 5: THE EFFECT OF STRESS RATIO R ON CRACK GROWTH

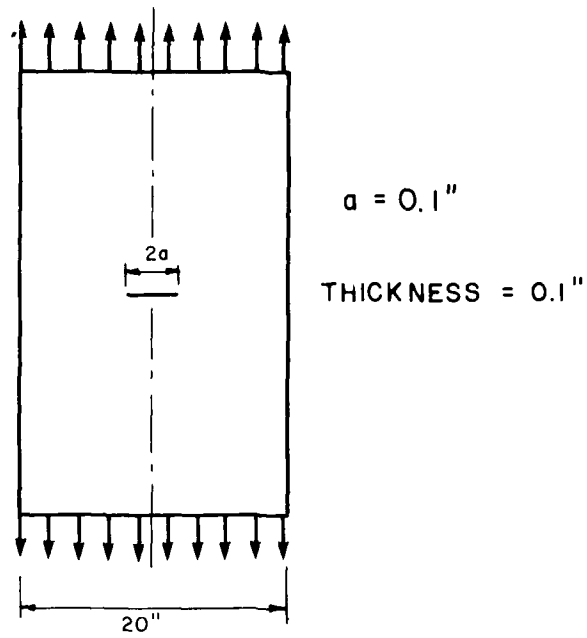


FIG. 6: A THROUGH CRACK

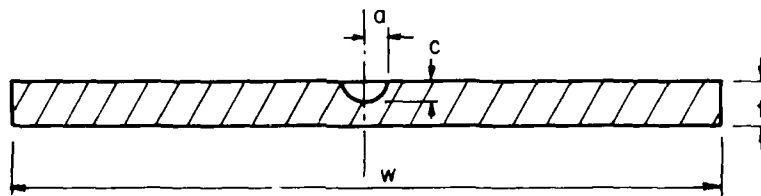


FIG. 7: A SURFACE CRACK

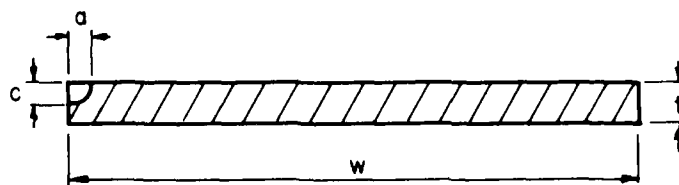


FIG. 8: A CORNER CRACK

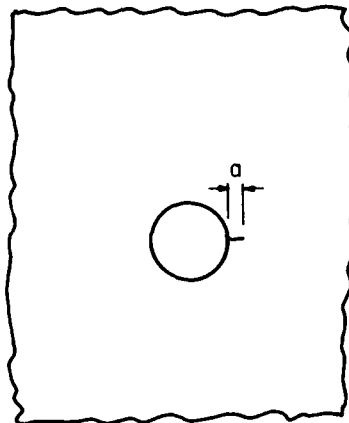


FIG. 9: A CRACK FROM A HOLE

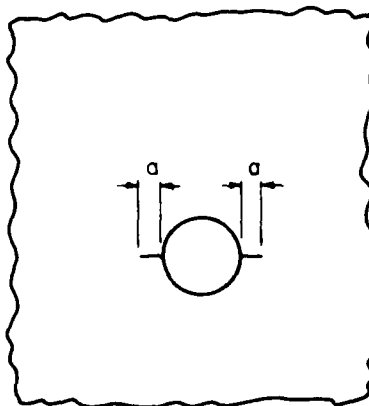


FIG. 10: A DOUBLE CRACK FROM A HOLE

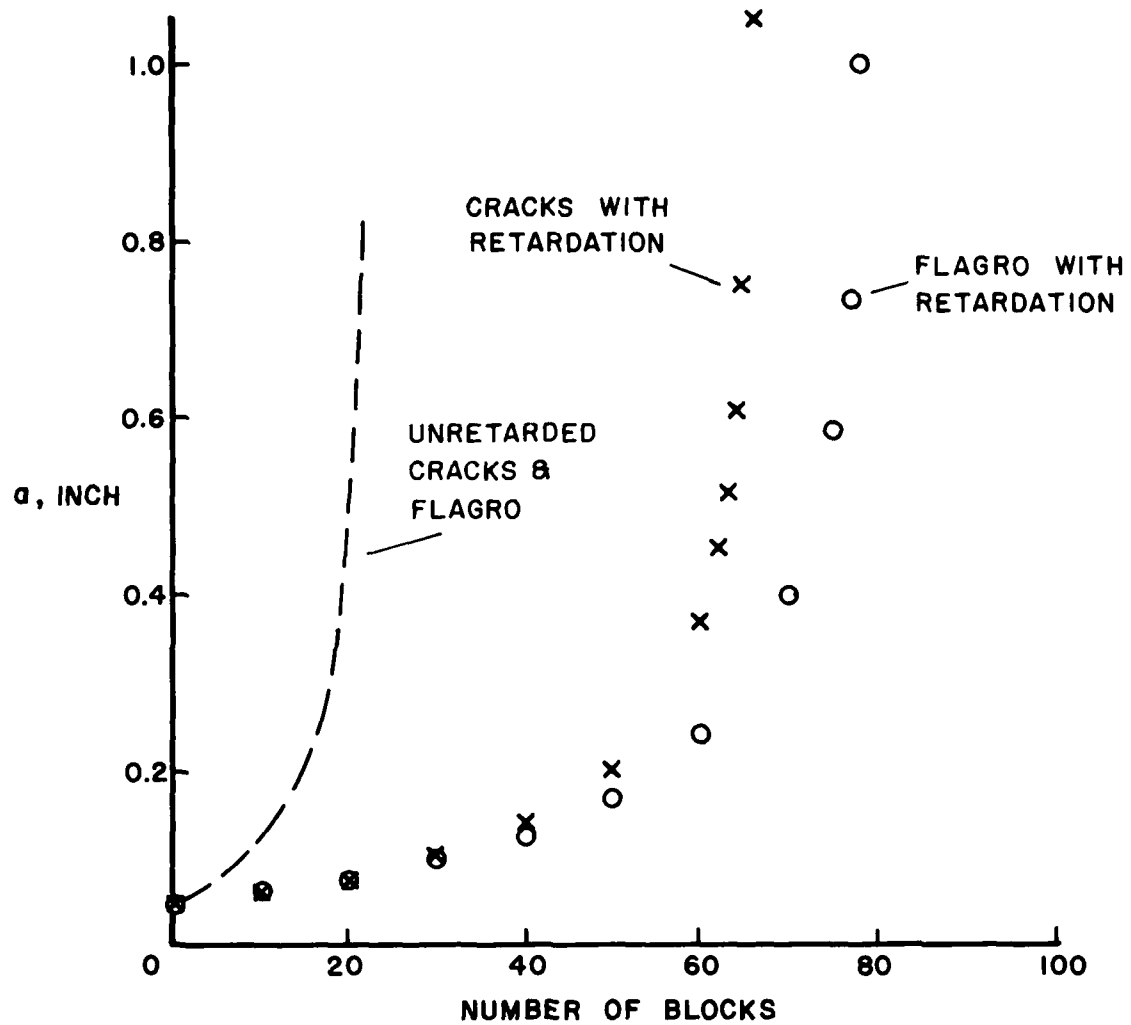


FIG. 11: A THROUGH CRACK IN ALUM 2219-T62 PLATE
FORMAN GROWTH WITH WHEELER RETARDATION

APPENDIX A

Crack growth models

- a. Paris (Ref. 4)

$$\frac{da}{dN} = C(\Delta K)^n$$

- b. Forman (Ref. 5)

$$\frac{da}{dN} = C \frac{(\Delta K)^n}{(1-R)K_c - \Delta K}$$

where K_c is fracture toughness

- c. Walker (Ref. 3)

$$\frac{da}{dN} = C \left[\frac{\Delta K}{(1-R)^{(1-m)}} \right]^n$$

- d. Collipriest (Ref. 6)

$$\frac{da}{dN} = C (K_c \Delta K_0)^{n/2} \cdot$$

$$\text{EXP} \left[\ln (K_c / \Delta K_0)^{n/2} \operatorname{arctanh} \frac{\ln[\Delta K^2 / \{(1-R)K_c \Delta K_0\}]}{\ln[(1-R)K_c / \Delta K_0]} \right]$$

where K_c is fracture toughness and ΔK_0 is the threshold stress intensity range.

